Base Pressure in Supersonic Flow: Further Thoughts About a Theory

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Introduction

IN 1988, the present author gave a very brief presentation of the essentials of his theory for the prediction of the base pressure in two-dimensional supersonic flow.[1] Thereafter, this theory was criticized by Magi and Gai[2] in 1988. Since in our opinion this critique is partly incorrect, we will in the following give a short contribution commenting on the critique by Magi and Gai[2] and simultaneously give further theoretical and experimental results concerning the base pressure in supersonic flow.

Base Pressure Calculations by Magi and Gai

In Fig. 1 of Ref. 2, the base pressure coefficient is presented as a function of the Mach number. The experimental results, as compiled by Nash [3] and shown in Fig. 1 of Ref. 2, are valid for a very thin boundary layer. Nash calls this base pressure the limiting base pressure.[3]

This value is considerably lower than the experimental base pressure coefficient, as shown in Fig. 7 of Nash.[3] As Magi and Gai[2] use this unrealistically small base pressure in the calculations, their predicted values for the quantity H^*/H cannot be compared with values that would be valid for a finite boundary-layer thickness. (H^* is the height of the shear layer affected wake shock and H is the reattaching shear layer thickness, see Ref. 1.) Moreover, the prediction of H^*/H from flow visualization gives uncertain values for this quantity. Magi and Gai[2] state that H^*/H should, at higher Mach numbers, be of the order of unity. This is not the case in the opinion of the present author.

New Theoretical and Experimental Results and Discussion

In Fig. 1, Fig. 7 of Ref. 3 is reproduced including a theoretical curve of Tanner by using $H^*/H = 7.37$ in the calculations. The experimental values of the base pressure coefficient compiled by Nash agree very will this theoretical curve for $\delta_2/d = 0.003$, where δ_2 is the boundary-layer momentum thickness. Therefore, $H^*/H = 7.37$ is a realistic value for this ratio at high Mach numbers. Of course, we agree with Magi and Gai[2] that the lip shock is important. The question is only if it is possible to consider its influence in simple general equations. Therefore, it is perhaps preferable to compute the H^*/H values using experimental results for the base pressure, as has been done by Tanner[4] in 1984. As shown in Fig. 1, the theory then agrees well with experiments.

In Fig. 2, further results are shown as obtained from measurements in the Ludwieg-Tube at Göttingen.[5] The experimental results for the extended ogive are compared with a theoretical curve for a 30-deg wedge with plate, then also for the ogive the wedge angle at the leading edge is about 30 deg. The theoretical values for the base pressure coefficient using $\delta_2/d = 0.045$ in the calculations are in good agreement with the experimental data. (The theoretical results were provided by Klaus-Dieter Droste, which is sincerely appreciated.)

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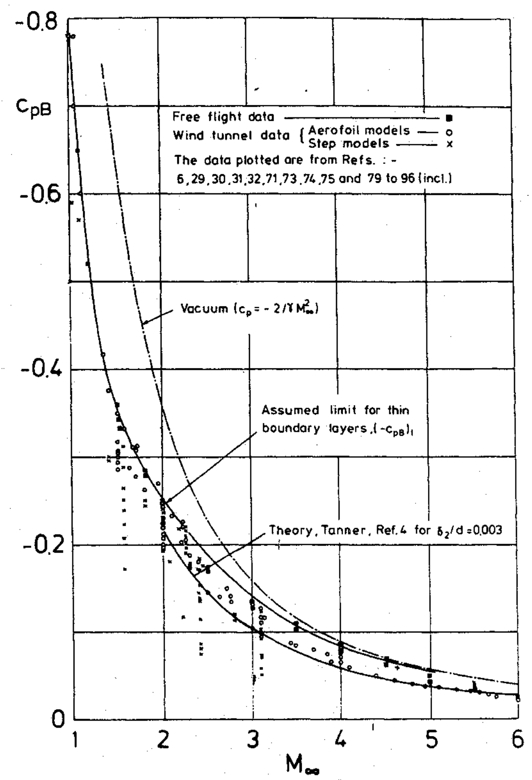

Fig. 1 Base pressure coefficient as a function of the Mach number (experimental results from Nash,[3] theory from Tanner[4] for $\delta_2/d = 0.003$).

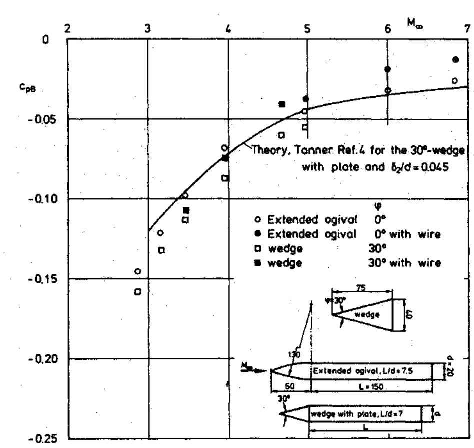

Fig. 2 Base pressure coefficient as a function of the Mach number (experimental results from Tanner,[5] theory from Tanner[4] for $\delta_2/d = 0.045$.)

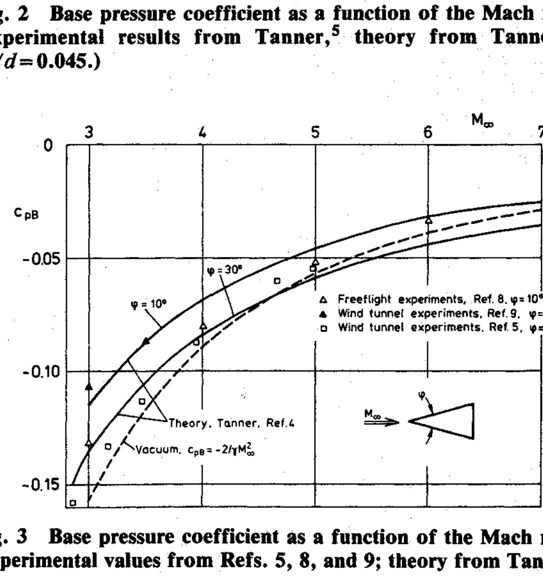

Fig. 3 Base pressure coefficient as a function of the Mach number (experimental values from Refs. 5, 8, and 9; theory from Tanner[4]).

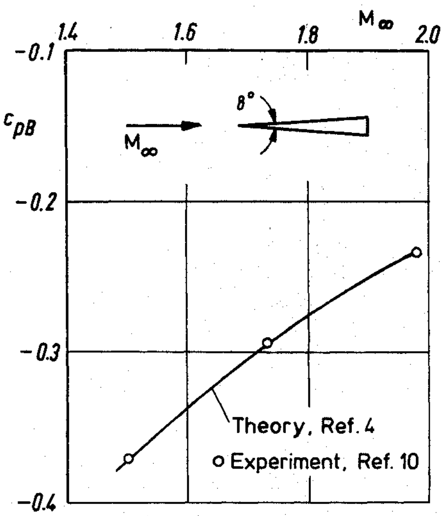

Fig. 4 Base pressure coefficient as a function of the Mach number (theory from Tanner,[4] experimental results from Tanner[10]).

For incompressible flow, the value $\delta_2/d = 0.040$ was obtained by methods given by Schlichting[6] with the ratio δ_2/d as defined by Eqs. (21.8) and (21.9) in Ref. 6. As stated in Ref. 6, the boundary-layer thickness δ increases rapidly with increasing Mach number. Since, simultaneously, the ratio δ_2/d becomes smaller, it seems plausible to assume that δ_2/d increases only moderately with increasing Mach number. For our calculations, $\delta_2/d = 0.045$ was used, which is 12.5% greater than the value $\delta_2/d = 0.040$ valid for incompressible flow.

The experimental base pressure coefficients for the 30-deg wedge are smaller than for the extended ogive. In both cases, the use of a thick wire ($d = 10$ mm) in front of the leading edge of the model causes a remarkable increase of the base pressure. This is an influence of the boundary-layer thickness because the base pressure increases with increasing boundary-layer thickness, as already shown in Ref. 1.

In Fig. 3, c_{pB} is plotted as a function of the freestream Mach number. The theoretical curves calculated by equations given in Ref. 4 agree very well with experimental values of the base pressure coefficient. For the 10-deg wedge, the wind-tunnel experiments of Ref. 9 agree well with the theoretical curve, whereas the C_{pB} values obtained in free flight are considerably lower, except at $M_\infty = 6$, where theory and experiment agree. For the 30-deg wedge, the theoretical C_{pB} values are for the Mach number $M_\infty > 4.6$, somewhat lower than vacuum. The theoretical base pressure is therefore negative, which is absurd. The reason for this anomaly is, at present, obscure. For $M_\infty \leq 4.0$, theoretical and experimental base pressure coefficients obtained for the 30-deg wedge are in good agreement, however.

In Fig. 4, results are shown for the 8-deg wedge in the Mach number range $M_\infty = 1.5$–2.0. In this case, the agreement between theory and experiment is excellent.

Conclusions

Magi and Gai[2] claim that the value $H^*/H = 7.37$ as used by the present author is valid only at low supersonic Mach numbers and that H^*/H should decrease with increasing Mach number. In their opinion, the value of H^*/H should be of the order of unity for Mach numbers $M_\infty > 3.5$ (see Fig. 2 of Ref. 2).

The new results presented here in Figs. 1–4 show, however, that using the value $H^*/H = 7.37$ at all Mach numbers gives theoretical results, which agree very well with experimentally observed base pressure coefficients for Mach numbers up to $M_\infty = 7$. Therefore, the value $H^*/H = 7.37$ seems to be universally applicable, contrary to the claims of Magi and Gai.[2]

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Numerical Investigation of Unsteady Transonic Nozzle Flows

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Introduction

UNSTEADY flow phenomena have frequently occurred in many places during aircraft flight such as flutter on wings caused by the interaction of elastic, inertial, and aerodynamic forces, buffeting at airfoil trailing edge due to strong shock/boundary-layer interaction, and buzz in inlets. The avoidance of these unwanted, unsteady flow phenomena through understanding their flow structures would be beneficial for aircraft design.

In this study, unsteady transonic nozzle flows with shock waves are considered, resulting from a fluctuating backpressure. In the past, unsteady nozzle flows with shocks has been studied by several investigators.[1–4] In Ref. 1, Richey and Adamson found that in the slowly time-varying regime, the amplitude of shock oscillation is $\mathcal{O}(\epsilon)$ if the imposed pressure fluctuation has an amplitude of $\mathcal{O}(\epsilon^2)$ and period of order ϵ^{-1}, where ϵ denotes a small parameter used to measure the difference between the flow velocity and the sound speed. Thus, the shock oscillation is linearly related to the imposed pressure fluctuation. In their study, the wall thickness was assumed to be $\mathcal{O}(\epsilon^2)$. However, in the present study it is found that the wall

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